**Fall 2020 CS 2510**

LAST NAME \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_FIRST NAME \_\_\_\_\_\_\_\_\_\_\_\_

**Homework 4, part 1**

Due on iCollege – part1.   
Part 2 is for 18 pts available on zyBooks, due October 27th 9am.

**PROBLEM 1.** [points:20] Using mathematical induction prove that n3- n is divisible by 6 for any non-negative integer n.

Hint: follow 3 steps of math induction. Also use your knowledge of how to prove things – what does it mean to be divisible by 6?

1.Basis step. [points: 5]

Let be the statement

Base case: .

, 6 divides when

So, this equation is true when .

2. Inductive hypothesis. [points: 5]

We should show that

If is true for some , it should also be true for another .

or

3. Proof. [points: 10]

For every 2 consecutive integers and , exactly one is a one of them is even, which implies that and consequently that . For a number to be divisible by , it will need to be divisible by both and . Since the base case proved that can be divisible by , we need not worry about it in this part of the sum. We know that because there is a multiple of 3 in the term; which implies that . This implies that . Since , we have shown that that , which is the inductive statement.

Therefore, . is divisible by .

**PROBLEM 2.** [points:30] Give a *recursive* definition of the sequence {an}, n = 1, 2, 3, … if

Hint: write down several members of sequences, and then express an through previous member(s). Don’t forget to add initial condition(s).

1. an = 4n – 2

The value of is

Now, I will add and subtract 4 to redefine this relationship:

The recursive definition is: , when and .

**b)** an = n(n+1)

The recursive definition is:

where and .

**c)** an = 4

where and

**PROBLEM 3** [points: 32]

Do **expand-guess-verify** technique for the following relationships, show step by step.

Find closed-form formula for these recursive relationships. Estimate running time complexity (Big Oh) of the closed-form formulas – those also give you idea how “good” or “bad” original recursive algorithms are!

f(1) = 5  
f(n) = f(n-1) + 4

When and ,

Sum =

Assuming

Show that

O(3n) = , as we exclude leading coefficients.

f(1) = 2  
f(n) = 3f(n-1)

When and :

This is a geometric series:

Sum of this series:

Sum:

For n = 1:

Sum:

Assuming that

Show that S(n+1) =

if we exclude leading constants.